MA 486-1E (Statistics), Dr Chernov — Midterm test #2

Show your work. — Fri, Mar 12, 2004

1. (10 pts) A manufacturer of automatic washers offers a particular model in one of three colors: white, yellow, blue. Of the first 1000 washers sold, 400 were white. Would you conclude that customers have preference for the white color? Justify your answer. Find the p-value of the test.

Solution: we are testing the hypothesis $H_0: \ p = 1/3$ against $H_1: \ p > 1/3$. The test for proportions gives

$$Z = \frac{0.4 - 0.3333}{\sqrt{0.3333 \times 0.6667/1000}} = 4.47.$$

The p-value of the test is

$$p = 1 - \Phi(4.4765) \approx 0.$$

Yes, the customers prefer the white color.

2. (20 pts) Fit a straight line to the five data points in the accompanying table:

x	-2	-1	0	1	2
y	3	2	1	-1	-1

Give estimates of α, β and σ^2. Draw a scatter plot.

Answers: $\bar{x} = 0$, $\bar{y} = 0.8$, $s_x^2 = 2.5$, $s_y^2 = 3.2$, $c_{xy} = -2.75$.
Now $\hat{\alpha} = 0.8$, $\hat{\beta} = -1.1$, and $\hat{\sigma}^2 = 0.14$.

3. (20 pts) An experiment is carried out to see if there is any relation between a man's age and whether he wears a moustache. The following table summarizes the results of the experiment:

age (in years)	18–30	31-45	over 45
wears a moustache	21	18	11
does not wear a moustache	79	82	89

Test the hypothesis that there is no relation between a man's age and whether he wears a moustache (assume $\alpha = 5\%$). What is the approximate p-value of the test?

Answers: $Q = 3.8 < \chi^2_{0.05}(2) = 5.991$, hence we accept H_0: there is no relation between a man's age and whether he wears a moustache. The p-value is between 0.1 and 0.9.

4. (20 pts) The closing prices of two common stocks were recorded for a period of 16 days. The sample means and the sample standard deviations were

$$\bar{x} = 400.24 \qquad \bar{y} = 412.18$$
$$s_x = 2.11 \qquad s_y = 3.93$$

Assume that the price of each stock is a normal random variable and test the hypothesis that the corresponding distributions are equal. Let $\alpha = 10\%$.

Solution: first we test the hypothesis that the variances are equal:

$$\frac{s_x^2}{s_y^2} = 0.288 < F_{0.05}(15, 15) = 2.4, \quad \frac{s_y^2}{s_x^2} = 3.47 > F_{0.05}(15, 15) = 2.4,$$

so we accept H_1: the distributions are different. There is no need to compare the mean values now.

5. (30 pts) Let X be $N(\mu, 25)$. We need to test the hypothesis $H_0 : \mu = 5$ against $H_1 : \mu > 5$. Let the sample size be $n = 100$.

(a) Let the critical region be $\bar{x} > 6.3$. Find α and write down formulas for β and $K(\mu)$.

Answer:

$$\alpha = 1 - \Phi\left(\frac{6.3 - 5}{5/10}\right) = 0.0047$$

$$\beta = \Phi\left(\frac{6.3 - \mu}{5/10}\right), \qquad K(\mu) = 1 - \beta$$

(b) Compute $K(6)$, $K(6.5)$ and $K(7)$. Sketch the graph of the power function.

Answers: $K(6) = 0.2743$, $K(6.5) = 0.6554$, $K(7) = 0.9192$.

(c) Assume that the experiment yields $\bar{x} = 6.1$. What is the p-value of the test?

Answer:

$$\text{p-value } = \Phi\left(\frac{6.1 - 5}{5/10}\right) = 0.0139$$

(d) Suppose we want $\alpha = 0.002$ and $K(6) = 0.995$. How large a sample will be necessary?

Answer:

$$n = \frac{25 \times (2.88 + 2.576)^2}{(6 - 5)^2} = 744$$

[Bonus] A sample $x_1, \ldots, x_{201}$ from $N(\mu, \sigma^2)$ yields $\bar{x} = 155$ and $s^2 = 10.1$. Test the hypothesis $H_0 : \sigma = 3$ against $H_1 : \sigma \neq 3$ at the 5% significance level.

Note: the sample size $n = 201$ exceeds the maximum number of degrees of freedom provided in Table IV. Thus you need to use normal approximation to χ^2.

Answer: the test statistic is

$$\chi^2 = \frac{200 \times 10.1}{9} = 224.4$$

The critical region consists of two intervals:

$$\chi^2 > \chi^2_{0.025}(200) \qquad \text{and} \qquad \chi^2 < \chi^2_{0.975}(200)$$

We use normal approximation to χ^2:

$$\chi^2(200) \approx N(200, 400) = 200 + 20Z$$

hence

$$\chi^2_{0.025}(200) = 200 + 20z_{0.025} = 239.2$$

and

$$\chi^2_{0.975}(200) = 200 + 20z_{0.975} = 160.8$$

Since $160.8 < 224.4 < 239.2$, we accept H_0.