MA 486-4A (Statistics), Dr Chernov — Midterm test #2

Every problem is 20 points. Show your work. — Thu, Mar 10, 2005

1. In a biology laboratory students test the Mendelian theory of inheritance using corn. The Mendelian theory claims that the frequencies of the four categories smooth and yellow, wrinkled and yellow, smooth and purple, and wrinkled and purple will occur in the ratio 9:3:3:1. If a student counted 277, 82, 95, and 26, respectively, for these four categories, would these data support the Mendelian theory? Let $\alpha = 0.01$.

Answers: $Q = 1.7$. Since $Q < \chi_{0.01}(3) = 11.34$, we accept H_0 (the data support the Mendelian theory).

2. Let X_1, X_2, X_3, X_4 be four independent random variables that have normal distributions $N(\mu_i, \sigma^2)$. Test the hypothesis

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$$

at the level $\alpha = 2.5\%$. The observed data are given in the table below:

X_1 :	12	10	9	13
X_2 :	9	8	7	12
X_3 :	13	8	12	11
X_4 :	6	8	12	10

Compute $SS(E)$, $SS(T)$, and use the F-test. Compose the ANOVA table (without the p-value).

Answers: $SS(E) = 58$, $SS(T) = 16$.
The ANOVA table is:

	SS	DoF	MS	F
Treatment	16	3	16/3	
Error	58	12	58/12	1.10
Total	74	15	74/15	

Since $1.1 < F_{0.025}(3, 12) = 4.47$, we accept H_0.

3. Let p_m and p_f be the respective proportions of male and female smokers in the US. In a random sample of 500 men, there were 85 smokers, and in a random sample of 200 women, there were 32 smokers. Test the hypothesis $H_0 : p_m = p_f$ against the alternative $H_1 : p_m \neq p_f$ at 4% level. Find the p-value of the test (up to 4 significant digits!).

Answers: $\hat{p} = 117/700 = 0.1671$ and $Z = 0.32$. Since $|Z| < z_{0.02} = 2.054$, we accept H_0.

The p-value is $2(1 - \Phi(0.32)) = 0.749$.

4. In the following table, 100 individuals are classified by gender and by whether they answer Yes, No, or Not Sure in a certain poll. Test the null hypothesis that the male and female responders tend to answer similarly (i.e. the probabilities of Yes, No, and Not Sure for males are the same as for females). Let $\alpha = 0.025$.

Gender	Yes	No	Not Sure
Male	12	32	12
Female	10	28	6

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Answers: $Q = 1.023$. Since $1.023 < \chi^2_{0.025}(2) = 7.378$, we accept H_0.

5. Let $x_1, \ldots, x_{11}$ and $y_1, \ldots, y_{16}$ be two independent samples taken from $N(\mu_x, \sigma_x^2)$ and $N(\mu_y, \sigma_y^2)$, respectively. The test results are $\bar{x} = -2$, $s_x = 5$, $\bar{y} = 3$ and $s_y = 6$.

Test the hypothesis of equality of these two normal random variables at the level $\alpha = 5\%$. Find an approximate p-value of the second part of the test (about the means).

Answers: for the test about variances: $F = 36/25 = 1.44$. Since $1.44 < F_{0.025}(15, 10) = 3.52$ and $1.44 > 1/F_{0.025}(10, 15) = 1/3.06$, we accept H_0.

For the test about means: $T = -2.27$. Since $|-2.27| > t_{0.025}(25) = 2.060$, we reject H_0 and accept H_1.

The p-value is between 0.025 and 0.05. (By the on-line calculator, it is 0.032.)

[Bonus] Let X be $N(\mu, 100)$. We need to test the hypothesis $H_0 : \mu = 10$ against $H_1 : \mu < 10$. Let the sample size be $n = 400$.
(a) Let the critical region be $\bar{x} < 9.2$. Find α and write down formulas for β and $K(\mu)$.

Answers: $\alpha = \Phi(-1.6) = 0.0548$;

$$\beta = \Phi\left(\frac{9.2 - \mu}{10/20}\right)$$

and $K(\mu) = 1 - \beta$.

(b) Compute $K(9.2)$, $K(9.4)$, $K(9.6)$ and $K(9.8)$. Sketch the graph of the power function.

Answer: $K(9.2) = 0.5$, $K(9.4) = 0.6554$, $K(9.6) = 0.7881$, and $K(9.8) = 0.8849$.

(c) Assume that the experiment yields $\bar{x} = 9.3$. What is the p-value of the test?

Answer: The p-value is $\Phi(-1.4) = 0.0808$.

(d) Suppose we want $\alpha = 0.002$ and $K(8.8) = 0.999$. How large a sample will be necessary?

Answer: $n \geq 2475.06$, so $n \geq 2476$.