MA 486/586-1G (Statistics), Dr Chernov — Midterm test #2

Show your work. — Fri, Mar 9, 2007

1. (10 pts) In a biology laboratory students test the Mendelian theory of inheritance using corn. The Mendelian theory claims that the frequencies of the four categories smooth and yellow, wrinkled and yellow, smooth and purple, and wrinkled and purple will occur in the ratio 9:3:3:1. If a student counted 171, 57, 69, and 23, respectively, for these four categories, would these data support the Mendelian theory? Let $\alpha = 0.05$. What can you say about the p-value of the test?

Answer: $Q = 2.4$, and since $Q < \chi^2_{0.05}(3) = 7.815$, we accept H_0.

2. (10 pts) For a public opinion poll in a close presidential election, let p be the proportion of voters in favor of candidate A. How large a sample should be taken if we want the maximum error of the estimate of p to be ≤ 0.01 with 97.5% confidence?

Answer: $n \geq \frac{2.24^2}{4 \cdot 0.01^2} = 12544.$

3. (10 pts) Let X be $N(\mu, 25)$. We need to test the hypothesis $H_0 : \mu_0 = 6$ against $H_1 : \mu_1 = 4$. Let the critical region be $\bar{x} \leq c$. Find values for n and c so that $\alpha = 0.0025$ and $K(\mu_1) = 0.99$. Do not forget to find the values of *both* n and c.

Answer: $n \geq \frac{(2.81+2.325)^2 \cdot 25}{(4-6)^2} = 165.$

4. (30 pts) Suppose a sample $x_1, \ldots, x_{25}$ from a normal distribution $N(\mu, \sigma^2)$ yielded $\bar{x} = 12$ and $s = 5$. Assume $\alpha = 0.1$.

(a) Test the hypothesis $H_0 : \mu = 10$ against $H_1 : \mu > 10$.

(b) Test the hypothesis $H_0 : \sigma^2 = 16$ against $H_1 : \sigma^2 \neq 16$.

Answers:

(a) $T = 2$, and since $T > t_{0.1}(24) = 1.318$, we accept H_1.

(b) $\chi^2 = 37.5$, and since $\chi^2 > \chi^2_{0.05}(24) = 36.42$, we accept H_1.

5. (40 pts) Let $x_1, \ldots, x_n$ be independent values of a random variable X with probability density function

$$f(x;\theta) = \frac{1}{\sqrt{\pi\theta}} e^{-\frac{(x-\theta)^2}{\theta}} = \frac{1}{\sqrt{\pi\theta}} e^{-\frac{x^2}{\theta}+2x-\theta}$$

where $\theta > 0$ is an unknown parameter. Note: this is a normal random variable $X = N(\mu, \sigma^2)$ with $\mu = \theta$ and $\sigma^2 = \theta/2$. Hence $E(X) = \theta$ and $\mathrm{Var}(X) = \theta/2$.

(a) Find sufficient statistics.

(b) Find the maximum likelihood estimate for θ, and make sure that the MLE is a function of sufficient statistics.

(c) Find the Rao-Cramer lower bound on the variance of unbiased estimates of θ. (Note: you can compute $E(X^2) = \mathrm{Var}(X) + [E(X)]^2$ by using the above formulas for $E(X)$ and $\mathrm{Var}(X)$.)

Answers:

(a) The only sufficient statistic is $u = x_1^2 + \cdots + x_n^2$.

(b) $\hat{\theta} = -\frac{1}{4} + \sqrt{\frac{1}{16} + \frac{u}{n}}$.

(c) $\mathrm{Var}\,\hat{\theta} \geq \frac{2\theta^2 n}{4\theta+1}$.