MA 486-12 (Statistics), Chernov | Midterm test #2
Show your work. | Wed, May 5

1. It is claimed that 60% of doctors recommend a certain brand of pain relief medicine. A consumer group doubts this claim (they think the estimate is too high). It conducts a survey that shows that out of 250 doctors, 136 prescribe this brand of medicine.

(a) State the null hypothesis H_0 and the alternative hypothesis H_1.
(b) Find the critical region for the significance level $\alpha = 0.05$.
(c) Which hypothesis do you accept?
(d) [Bonus] Find an approximate p-value of the test.

Some answers:

$H_0:\ p = 0.6$ and $H_1:\ p < 0.6$

point estimate is $\hat{p} = 0.544$

p-value is 0.03

accept H_1.

2. Let $x_1, \ldots, x_{25}$ be taken from $N(\mu, 25)$. To test $H_0 : \mu = 30$ against $H_1 : \mu > 30$ let the critical region be $\bar{x} \geq 32$.

(a) Find formulas for $\alpha, \beta, K(\mu)$.
(b) Compute α and $K(31)$, $K(32)$, $K(33)$, $K(34)$. Sketch the graph of the power function.
(c) Let $\bar{x} = 32.6$. What is the p-value of the test?

Some answers:

$\alpha = 1 - \Phi(32 - 30) = 0.0228$

$\beta = \Phi((32 - \mu)/(5/5)) = \Phi(32 - \mu)$

$K = 1 - \Phi(32 - \mu)$

K's are 0.1587, 0.5, 0.8413, 0.9772

p-value is 0.0047

3. Let X be $N(\mu, 64)$. We need to test the hypethesis To test $H_0 : \mu_0 = 20$ against $H_1 : \mu_1 = 22$. Let the critical region be $\bar{x} \geq c$. Find values for n and c so that $\alpha = 0.025$ and $K(\mu_1) = 0.99$.

Answers:

Equations are

$$\frac{-c+22}{8/\sqrt{n}} = 2.33$$

and

$$\frac{c-20}{8/\sqrt{n}} = 1.96$$

Solutions are $n = 294$ and $c = 20.914$.

4. Let $x_1, \ldots, x_{10}$ be taken from $N(\mu, \sigma^2)$ to test $H_0 : \mu = 5$ against $H_1 : \mu \neq 5$. The test results are $\bar{x} = 4.3$ and $s^2 = 1.2$.

(a) Do we accept H_0 at the 10% significance level?
(b) What is the approximate p-value of the test?

Answers:

$$T = \frac{|4.3 - 5|}{\sqrt{1.2}/\sqrt{10}} = 2.02$$

This is greater than $t_{0.05}(9) = 1.833$, so accept H_1.

p-value is 0.08.

5. Let $x_1, \ldots, x_{20}$ be taken from $N(\mu, \sigma^2)$ to test $H_0 : \sigma = 8$ against $H_1 : \sigma > 8$. The test results are $\bar{x} = 6$ and $s^2 = 90$.

(a) Do we accept H_0 at the 5% significance level?
(b) What is the approximate p-value of the test?

Answers:

$\chi^2 = 26.72$. This is less than $\chi^2_{0.05}(19) = 30.14$, so accept H_0.

p-value is 0.13

6. Let $x_1, \ldots, x_{10}$ and $y_1, \ldots, y_{13}$ be two independent samples taken from $N(\mu_x, \sigma_x^2)$ and $N(\mu_y, \sigma_y^2)$, respectively. The test results are $\bar{x} = 6$, $s_x^2 = 90$, $\bar{y} = 11$ and $s_y^2 = 100$.

(a) Test the hypethesis $\sigma_x^2 = \sigma_y^2$ against $\sigma_x^2 \neq \sigma_y^2$. Let $\alpha = 10\%$.
(b) Assume $\sigma_x^2 = \sigma_y^2$, and test the hypothesis $\mu_x = \mu_y$ against $\mu_x \neq \mu_y$. Let $\alpha = 5\%$.
(c) What is the approximate p-value of the test in part (b)?
(d) [Bonus] Despite the fact that $\sigma_x^2 = \sigma_y^2$ is accepted in part (a), let us say we suspect the equality is not valid. Thus use the test proposed by Welsh to test $\mu_x = \mu_y$ against $\mu_x \neq \mu_y$. Let $\alpha = 5\%$.

Answers:

(a) $90/100 = 0.9$ is less than $F_{0.05}(9, 12) = 2.80$ and $100/90 = 1.111$ is less than $F_{0.05}(12, 9) = 3.07$, so accept H_0

(b) test statistic

$$T = \frac{6 - 11}{\sqrt{\frac{9.90+12.10}{21}}\sqrt{\frac{1}{10} + \frac{1}{13}}} = -1.215$$

$t_{\alpha/2}(21) = 2.080$

(c) p-value is 0.22

(d) test statistic

$$T = \frac{-5}{\sqrt{\frac{90}{10} + \frac{100}{13}}} = -1.2238$$

$t_{0.025}(20) = 2.086$. Accept H_0.

7. In a biology laboratory students test the Mendelian theory of inheritance using corn. The Mendelian theory claims that the frequencies of the four categories smooth and yellow, wrinkled and yellow, smooth and purple, and wrinkled and purple will occur in the ratio 9:3:3:1. If a student counted 459, 130,149, and 62, respectively, for these four categories, would these data support the Mednelian theory? Let $\alpha = 0.1$.

For extra credit, find the approximate p-value of the test.

Answers:

$Q = 5.733$, $\chi^2_{0.1}(3) = 6.251$. Accept H_0.

p-value is 0.13

8. In the following table, 1000 individuals are classified by gender and by whether they answer Yes, No, or Not Sure in a certain poll. Test the null hypothesis that the gender and the answers are independent. Let $\alpha = 0.05$. Find the approximate p-value of the test.

Gender	Yes	No	Not Sure	Total
Male	340	255	45	640
Female	175	145	40	360

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Answers:

$Q = 5.434$, $\chi^2_\alpha(2) = 5.991$. Accept H_0.

p-value is 0.07