1. How many rearrangements of the word STATISTICAL are there? Answer: $\frac{11!}{3!\,2!\,2!\,2!} = 831600$

2. How many solutions exist to the equation $x_1 + x_2 + x_3 = 12$, where x_1, x_2, and x_3 have to be non-negative integers? [Note: the solution $x_1 = 9$, $x_2 = 2$, $x_3 = 1$ is not the same as $x_1 = 1$, $x_2 = 2$, $x_3 = 9$.] Answer: $\binom{12+3-1}{3-1} = 91$

3. You are given $P(A \cup B) = 0.6$ and $P(A \cup B^c) = 0.8$. Determine $P(A)$. Justify your answer. Answer: 0.4

4. If the probability is 0.3 that a child exposed to a certain contagious disease will catch it, what is the probability that the eighth child exposed to the disease will be the second to catch it? Answer: $\binom{7}{1} \cdot 0.3^2 \cdot 0.7^6 \approx 0.074$

5. There are 2,020 pickup trucks in Jasper, AL. Of these, 12 are stolen. Suppose that 10 randomly chosen pickups are checked by the police. What is the probability that either 1 or 2 of the 10 chosen pickups are stolen? Give the expression without finding the exact numeric value. Answer: $P(X = 1) + P(X = 2) = \frac{C_{12,1}C_{2020-12,10-1}+C_{12,2}C_{2020-12,10-2}}{C_{2020,10}}$

6. A random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{1}{2}\sqrt{x} & \text{for } 0 < x < 1 \\ 0.6 & \text{for } 1 < x < 2 \\ \frac{2}{5}x - \frac{1}{5} & \text{for } 2 < x < 3 \\ 1 & \text{for } x > 3 \end{cases}$$

(a) What is the probability that $X = 1$? Answer: 0.1
(b) What is the value of $F(1)$? Answer: 0.6
(c) What is the probability that $0.25 < X < 2.5$? Answer: 0.55
(d) Is X discrete? Continuous? A mixture of the two? Justify your answer. Answer: a mixture of the two
(Bonus) Find the mean value of X. Answer: ≈ 1.267

7. The lifetime of a printer costing \$100 is exponentially distributed with mean 5 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first or second year following its purchase, and a one-half refund if it fails during the third or fourth year. If the manufacturer sells 10 printers, how much should he expect to pay in refunds? Answer: \$440 (approx.)

8. Let X be a normal random variable with mean 2 and variance 4. Find $P(X^2 > 4X+5)$. Answer: 0.1336

9. A fisherman whose average time for catching a fish is 10 minutes wants to bring home exactly 2 fishes. What is the probability he will need between 10 and 20 minutes to catch them? Answer: 0.3298

10. Consider the following hypothetical situation. Grade data indicates that on the average 80% of the students in senior engineering classes have passed. There is variation among classes, however, and the proportion X must be considered a random variable. From past data we have measured a variance of $\frac{2}{75}$. We would like to model the proportion X of passing students with a Beta distribution.
(a) Find the density function of X. Answer: $f(x) = 4x^3$ for $0 < x < 1$
(b) Find the probability that more than 90% of the students passed. Answer: 0.3439
(Bonus) Calculate the difference between the 25th and 75th percentiles of X. Answer: 0.2235