MA 631-14 (Applied linear algebra), Dr. Chernov — Final exam

Justify your work. — Mon, November 24

1. Let $B = \{(1,1,1),(2,1,1,),(1,2,0)\}$ be a basis in $\mathbb{R}^3$. Find the transition matrices $P_{B,E}$ and $P_{E,B}$, where E stands for the standard basis in $\mathbb{R}^3$. Find a matrix A such that $Av = (v)_B$ for all $v \in \mathbb{R}^3$.

2. Let W be the subspace of the differentiable functions on the real line with basis $B = \{\sin^2 x, \sin x \cos x, \cos^2 x\}$. Let T be the linear transformation defined by $Tf = f'$.
(a) Show that W is T-invariant. Is $T : W \to W$ a bijection?
(b) Is $T : W \to W$ diagonalizable over the real field?
(c) Is $T : W \to W$ diagonalizable over the complex field?

3. Prove that if two matrices, A and B, are similar, then their ranks are equal.

4. Let $W := \operatorname{span}\{(3,1,1,3),(1,2,2,1),(0,1,1,0)\}$ in $\mathbb{R}^4$.
(a) Find $\dim W$ and a basis of W.
(b) Find a basis of $W^\perp$ in $\mathbb{R}^4$ with the standard inner product.

5. Let $v_1 = (-1,0,1)$, $v_2 = (2,-2,0)$, $v_3 = (0,-1,-1)$ in $\mathbb{R}^3$.
(a) Find an ONB $\{w_1, w_2, w_3\}$ of $\mathbb{R}^3$ by using the Gram-Schmidt procedure.
(b) Find c_1, c_2, c_3 such that $c_1w_1 + c_2w_2 + c_3w_3 = (0,0,2)$.

6. Find a Jordan canonical form for

$$A = \begin{pmatrix} 1 & 1 & 0 & -1 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

7. (a) Let A be a 13×13 complex matrix with $C_A(x) = x^7(x-i)^6$, $M_A(x) = x^4(x-i)^3$, and $\dim E_0 = 3$, $\dim E_i = 2$. Find a Jordan canonical form of A.
(b) Let A be a 6×6 complex matrix with $C_A(x) = (x^2+1)^3$, $\dim E_i = 2$ and $\dim E_{-i} = 1$. Find the minimal polynomial of A.

8. Let $\{w_1, \ldots, w_n\}$ be an ONB in $\mathbb{R}^n$. What is $||w_1 + \cdots + w_n||$? Assuming that n is even, what is $||w_1 - w_2 + w_3 - \cdots - w_n||$?