MA 660-2C (Numerical Linear Algebra), Dr. Chernov Justify your work. Do 3 out of 4 problems for full credit.

Midterm test Thu, October 18, 2001

1. (a) Show that  $\kappa(A) = \kappa(A^{-1})$ . (b) Show that  $\kappa(AB) \leq \kappa(A)\kappa(B)$ .

(Here  $\kappa$  can be defined by any norm.)

2. Find the condition numbers  $\kappa_1(A)$  and  $\kappa_2(A)$  for

$$A = \left(\begin{array}{cc} 2 & 1\\ 5 & 2 \end{array}\right)$$

3. Find a QR-decomposition for

$$A = \left(\begin{array}{rr} 1 & 3\\ 1 & 2 \end{array}\right)$$

Make sure the diagonal elements of R are positive.

4. Prove that the matrix

$$A = \left(\begin{array}{cc} 5 & 2\\ 2 & 1 \end{array}\right)$$

is positive definite in two ways. First, use Sylvester's theorem. Second, find the Cholesky factorization for A.