MA 660-2C, Dr. Chernov

Computer assignment # 1

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Due Dec 6

1. Write a program to compute the Cholesky decomposition of a positive definite matrix A. Your program should be written in the form of a subroutine that takes the matrix A (and its dimension) as input and returns it overwritten by the Cholesky factor G. Be sure to include in the arguments to your subroutine a flag variable that can be set if the matrix does not have a Cholesky factor (i.e. it is not a positive definite matrix). Note that, as A is symmetric, you only need provide storage for the lower half of A. The return matrix G is lower triangular, so it is enough to compute its lower half only.

Try out your program on some matrices, make sure it works properly.

2. Write a main program for solving systems of linear equations $Ax = b$ with a positive definite matrix A using your subroutine for Cholesky decomposition and subroutines for forward and backward substitutions.

Use your program to solve $Ax = b$ for the matrix $A = (a_{ij})$, $1 \le i, j \le n$, where

$$a_{ij} = \begin{cases} 2, & \text{if } \ i = j \\ -1, & \text{if } \ i = j - 1 \ \text{ or } \ i = j + 1 \\ 0, & \text{otherwise} \end{cases}$$

and $b = (b_i)$, $1 \le i \le n$, where

$$b_i = \left(\frac{i}{n+1} \right)^4$$

Try different values for n.

The above system is a finite difference approximation of the boundary value problem

$$-x''(t) = t^4, \quad 0 < x < 1, \quad x(0) = 0, \quad x(1) = 0$$

Solve this problem directly and compare the solution with the vector x found by $Ax = b$. How does the error vary with n?