

1. Let $\{w_1, \dots, w_n\}$ be an ONB in \mathbb{R}^n . Assuming that n is even, compute

$$\|w_1 - w_2 + w_3 - \dots + w_{n-1} - w_n\|.$$

2. Let V be an inner product space, and $\|\cdot\|$ be the norm induced by the inner product. Recall that the parallelogram law states

$$\|u + v\|^2 + \|u - v\|^2 = 2\|u\|^2 + 2\|v\|^2.$$

Based on this, show that the norms $\|\cdot\|_1$ and $\|\cdot\|_\infty$ in \mathbb{R}^2 are **not** induced by any inner product.

3. Prove the **Neuman lemma**: if $\|A\| < 1$, then $I - A$ is invertible. Here $\|\cdot\|$ is a norm on the space of $n \times n$ matrices induced by any norm on \mathbb{R}^n or \mathbb{C}^n .