MA 660-2C, Dr Chernov

Warm-up homework assignment #1

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Due Thu, Aug 30

1. Problem 3.1 from Textbook.

2. Problem 3.3 from Textbook.

3. Show that the norm $||A|| = \max_{i,j} |a_{ij}|$ on the space of $n \times n$ real matrices is not induced by any norm in $\mathbb{R}^n$. Hint: use 8.10(ii).

4. Prove the Neuman lemma: if $||A|| < 1$, then $I - A$ is invertible. Here $|| \cdot ||$ is a norm on the space of $n \times n$ matrices induced by a norm on $\mathbb{R}^n$ or $\mathbb{C}^n$.

5. Let $V$ be an inner product space, and $|| \cdot ||$ be the norm induced by the inner product. Prove the parallelogram law

$$||u+v||^2 + ||u-v||^2 = 2||u||^2 + 2||v||^2$$

Based on this, show that the norms $|| \cdot ||_1$ and $|| \cdot ||_\infty$ in $\mathbb{R}^2$ are not induced by any inner product.

Extra credit: Find two norms on the space $C[0,1]$ that are not equivalent.