MA 660-1D, Dr Chernov | Warm-up homework assignment #1
. | Due Fri, Jan 16

1. Show that the norm $||A|| = \max_{i,j} |a_{ij}|$ on the space of $n \times n$ real matrices is not induced by any norm in $\mathbb{R}^n$. Hint: use 8.10(ii).

2. Prove the Neuman lemma: if $||A|| < 1$, then $I - A$ is invertible. Here $||\cdot||$ is a norm on the space of $n \times n$ matrices induced by a norm on $\mathbb{R}^n$ or $\mathbb{C}^n$.

3. Let V be an inner product space, and $||\cdot||$ be the norm induced by the inner product. Prove the parallelogram law

$$||u+v||^2 + ||u-v||^2 = 2||u||^2 + 2||v||^2$$

Based on this, show that the norms $||\cdot||_1$ and $||\cdot||_\infty$ in $\mathbb{R}^2$ are not induced by any inner product.

4. Let $\{w_1, \dots, w_n\}$ be an ONB in $\mathbb{R}^n$. Assuming that n is even, compute

$$\|w_1 - w_2 + w_3 - \cdots - w_n\|$$

5. Apply Gram-Schmidt orthonormalization to the the basis $\{1, x, x^2\}$ in the space $P_2(\mathbb{R})$ with the inner product

$$\langle f, g\rangle = \int_0^1 f(x)g(x)\,dx$$

Extra credit: Find two norms on the space $C[0,1]$ that are not equivalent.