MA 660-3A, Dr Chernov — Assignment #1

. — Due Mon, Jan 14

1. Show that the norm $\|A\| = \max_{i,j} |a_{ij}|$ on the space of $n \times n$ real matrices is not induced by any norm in $\mathbb{R}^n$. Hint: use the last inequality in 1.5.

2. Prove the **Neuman lemma**: if $\|A\| < 1$, then $I - A$ is invertible. Here $\|\cdot\|$ is a norm on the space of $n \times n$ matrices induced by any norm on $\mathbb{R}^n$ or $\mathbb{C}^n$.

3. Let V be an inner product space, and $\|\cdot\|$ be the norm induced by the inner product. Prove the parallelogram law

$$\|u+v\|^2 + \|u-v\|^2 = 2\|u\|^2 + 2\|v\|^2.$$

Based on this, show that the norms $\|\cdot\|_1$ and $\|\cdot\|_\infty$ in $\mathbb{R}^2$ are not induced by any inner product.

4. Let $\{w_1, \dots, w_n\}$ be an ONB in $\mathbb{R}^n$. Assuming that n is even, compute

$$\|w_1 - w_2 + w_3 - \cdots + w_{n-1} - w_n\|.$$