MA 660-2C, Dr Chernov

Homework assignment #10

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Due Thu, Nov 8

1. (JPE, September 1996) Compute the singular values of

$$A = \begin{pmatrix} 0 & -1.6 & 0.6 \\ 0 & 1.2 & 0.8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2. (JPE, September 1997). Show that, given a matrix $A \in \mathbb{R}^{n\times n}$, one can choose vectors b and Δb so that if

$$Ax = b$$

$$A(x + \Delta x) = b + \Delta b$$

then

$$\frac{||\Delta x||_2}{||x||_2} = \kappa_2(A)\frac{||\Delta b||_2}{||b||_2}$$

Explain the significance of this result for the 'optimal' role of condition numbers in the sensitivity analysis of linear systems.
(Hint: use the SVD theorem to show that it is enough to consider the case where A is a diagonal matrix.)

3. Prove that full rank matrices make an open subset of $\mathbb{R}^{m\times n}$.

4. (JPE, September 1998). Show that diagonalizable (complex) matrices make a dense subset of $\mathbb{C}^{n\times n}$. That is, for any $A \in \mathbb{C}^{n\times n}$ and $\varepsilon > 0$ there is a diagonalizable $B \in \mathbb{C}^{n\times n}$ such that $||A - B||_2 < \varepsilon$. (Hint: use Schur decomposition theorem).

5. Find the numerical rank with tolerance 0.9 of the matrix

$$A = \begin{pmatrix} 3 & 2 \\ -4 & -5 \end{pmatrix}$$