MA 660-3A, Dr Chernov

Assignment #10

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Due Tue, Apr 1

1. JPE, September 2002). Consider a linear system $Ax = b$. Let x^* be the exact solution, and let x_c be some computed approximate solution. Let $e = x^* - x_c$ be the error and $r = b - Ax_c$ the residual for x_c. Show that

$$\frac{1}{\kappa(A)} \frac{\|r\|}{\|b\|} \leq \frac{\|e\|}{\|x^*\|} \leq \kappa(A) \frac{\|r\|}{\|b\|}$$

Interpret the above inequality for $\kappa(A)$ close to 1 and for $\kappa(A)$ large.

2. (JPE, May 1997). Compute the condition numbers κ_1, κ_2 and κ_∞ for the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 1.01 & 2 \end{pmatrix}$$

3. Let $x, y \in \mathbb{C}^n$ be such that $x \neq y$ and $\|x\|_2 = \|y\|_2 \neq 0$. Show that there is a unique reflector matrix P such that $Px = y$ if and only if $\langle x, y \rangle \in \mathbb{R}$.

Bonus (JPE, May 1992). Compute the condition number κ_∞ for the matrix

$$A_n = \begin{pmatrix} 1 & 2 \\ 2 & 4 + n^{-2} \end{pmatrix}$$

Now, suppose that the systems $A_n x = b$, $n = 1, 2, \ldots$, are being solved for some $b \in \mathbb{R}^2$ on a computer employing binary floating point arithmetic with a 23 digit mantissa, and using chopped arithmetic. For which values of n can the computed solution be trusted? (Hint: first write down the unit roundoff (machine precision) $\mathbf{u}$).