MA 660-3A, Dr Chernov

Assignment #10

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Due Mon, Mar 29

1. Let $x, y \in \mathbb{C}^n$ be such that $x \neq y$ and $\|x\|_2 = \|y\|_2 \neq 0$. Show that there is a unique reflector matrix P such that $Px = y$ if and only if $\langle x, y \rangle \in \mathbb{R}$.

Bonus (JPE, May 1992). Compute the condition number κ_∞ for the matrix

$$A_n = \begin{pmatrix} 1 & 2 \\ 2 & 4 + n^{-2} \end{pmatrix}$$

Now, suppose that the systems $A_n x = b$, $n = 1, 2, \ldots$, are being solved for some $b \in \mathbb{R}^2$ on a computer employing binary floating point arithmetic with a 23 digit mantissa, and using chopped arithmetic. For which values of n can the computed solution be trusted? (Hint: first write down the unit roundoff (machine precision) $\mathbf{u}$).