1. Let $x, y \in \mathbb{C}^n$ be such that $x \neq y$ and $\|x\|_2 = \|y\|_2 \neq 0$. Show that there is a unique reflector matrix P such that $Px = y$ if and only if $\langle x, y \rangle \in \mathbb{R}$.

2 (JPE, May 2010). Let
$$A = \begin{bmatrix} 3 & -3 \\ 0 & 4 \\ 4 & -1 \end{bmatrix}$$
Find the QR factorization of A by Householder reflectors.

3 (JPE, May 1992). Compute the condition number κ_∞ for the matrix
$$A_n = \begin{pmatrix} 1 & 2 \\ 2 & 4 + n^{-2} \end{pmatrix}$$

Now, suppose that the systems $A_n x = b$, $n = 1, 2, \ldots$, are being solved for some $b \in \mathbb{R}^2$ on a computer employing binary floating point arithmetic with a 23 digit mantissa, and using chopped arithmetic. For which values of n can the computed solution be trusted? (Hint: first write down the unit roundoff (machine precision) $\mathbf{u}$).