MA 660-2C, Dr Chernov — Homework assignment #11

. — Due Thu, Nov 15

1. Let $x, y \in \mathbb{C}^n$ and $A = yx^*$. Show that $||A||_2 = ||x||_2 \cdot ||y||_2$.

2. (JPE May, 1994). Let $X^{-1}AX = D$, where D is a diagonal matrix.
(i) Show that the columns of X are right eigenvectors and the conjugate rows of X^{-1} are left eigenvectors of A.
(ii) Let $\lambda_1 \ldots, \lambda_n$ be the eigenvalues of A. Show that there are right eigenvectors $x_1, \ldots, x_n$ and left eigenvectors $y_1, \ldots, y_n$ such that

$$A = \sum_{i=1}^{n} \lambda_i x_i y_i^*$$

3. Let $A \in \mathbb{C}^{n\times n}$ be Hermitean with eigenvalues $\lambda_1 \leq \cdots \leq \lambda_n$. Let $\mu_1 \leq \cdots \leq \mu_{n-1}$ be all the eigenvalues of the $(n-1)$-st principal minor A_{n-1} of A. Use the Minimax theorem to prove the *interlacing property*

$$\lambda_1 \leq \mu_1 \leq \lambda_2 \leq \cdots \leq \lambda_{n-1} \leq \mu_{n-1} \leq \lambda_n$$