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Assignment #11 Due Mon, Apr 16

1. (JPE, September 2009) Let $A \in \mathbb{C}^{n \times n}$ be nonsingular. Let $A = Q_1 R_1$ be a QR decomposition of A, and for $k \ge 1$ define inductively $AQ_k = Q_{k+1}R_{k+1}$, a QR decomposition of AQ_k .

(a) Prove that there exists an upper triangular matrix U_k such that $Q_k = A^k U_k$ and a lower triangular matrix L_k such that $Q_k = (A^*)^{-k} L_k$.

(b) Suppose $\lim_{k\to\infty} R_k = R_\infty$ and $\lim_{k\to\infty} Q_k = Q_\infty$ exist. Determine the eigenvalues of A in terms of R_∞ .

2. (JPE, May 2006) Let $A \in \mathbb{C}^{n \times n}$ be tri-diagonal and Hermitian, with all its superdiagonal entries nonzero. Prove that the eigenvalues of A are distinct.

(Hint: show that for any scalar λ , the matrix $A - \lambda I$ has rank at least n - 1.)