MA 660-2C, Dr Chernov | Homework assignment #12
. | Due Thu, Nov 29

1. Let $A \in \mathbb{C}^{n\times n}$. Show that
(i) λ is an eigenvalue of A iff $\bar{\lambda}$ is an eigenvalue of A^*.
(ii) if A is normal, then for each eigenvalue the left and right eigenspaces coincide;
(iii) if A is normal, then for any simple eigenvalue λ of A we have $K(\lambda) = 1$.

2. Let $A \in \mathbb{C}^{n\times n}$ and $B = Q^*AQ$, where Q is a unitary matrix. Show that if the left and right eigenspaces of A are equal, then B enjoys the same property. After that show that A is normal. Conclude that if A has all simple eigenvalues with $K(\lambda) = 1$, then A is normal.

3. If λ is an eigenvalue of geometric multiplicity ≥ 2 for a matrix A, show that for each right eigenvector x there is a left eigenvector y such that $y^*x = 0$.