MA 660-3A, Dr Chernov

Assignment #12

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Due Tue, Apr 15

1. $A \in \mathbb{C}^{n\times n}$ and $B = Q^*AQ$, where Q is a unitary matrix. Show that if the left and right eigenspaces of A are equal, then B enjoys the same property. After that show that A is normal. Conclude that if A has all simple eigenvalues with $K(\lambda) = 1$, then A is normal.

2. If λ is an eigenvalue of geometric multiplicity ≥ 2 for a matrix A, show that for each right eigenvector x there is a left eigenvector y such that $y^*x = 0$.

3. Use the Gershgorin theorem to show that a symmetric, strictly row diagonally dominant real matrix with positive diagonal elements is positive definite.