MA 660-3A, Dr Chernov — Assignment #12

. — Due Mon, Apr 12

1. Let $A \in \mathbb{C}^{n\times n}$. Show that
(i) λ is an eigenvalue of A iff $\bar{\lambda}$ is an eigenvalue of A^*.
(ii) if A is normal, then for each eigenvalue the left and right eigenspaces coincide;
(iii) if A is normal, then for any simple eigenvalue λ of A we have $K(\lambda) = 1$.

2. Let $A \in \mathbb{C}^{n\times n}$ and $B = Q^*AQ$, where Q is a unitary matrix. Show that if the left and right eigenspaces of A are equal, then B enjoys the same property. After that show that A is normal. Conclude that if A has all simple eigenvalues with $K(\lambda) = 1$, then A is normal.

3. Use the Gershgorin theorem to show that a symmetric, strictly row diagonally dominant real matrix with positive diagonal elements is positive definite.

4. (JPE, May 1989) Let A be a real symmetric matrix, with eigenvalues λ_i, $1 \le i \le n$, satisfying

$$|\lambda_1| > |\lambda_2| \ge \cdots \ge |\lambda_n|$$

If x_1 is an eigenvector corresponding to λ_1, and z_0 is a vector satisfying $z_0^t x_1 \ne 0$, prove that

$$\lim_{k\to\infty} \frac{z_0^t A^k z_0}{z_0^t A^{k-1} z_0} = \lambda_1$$