MA 660-2C, Dr Chernov — Homework assignment #13

. — Due Thu, Dec 6

1. Use the Gershgorin theorem to show that a symmetric, row diagonally dominant real matrix with positive diagonal elements is positive definite.

2. (JPE, May 1989) Let A be a real symmetric matrix, with eigenvalues λ_i, $1 \le i \le n$, satisfying

$$|\lambda_1| > |\lambda_2| \ge \cdots \ge |\lambda_n|$$

If x_1 is an eigenvector corresponding to λ_1, and z_0 is a vector satisfying $z_0^t x_1 \ne 0$, prove that

$$\lim_{k\to\infty} \frac{z_0^t A^k z_0}{z_0^t A^{k-1} z_0} = \lambda_1$$

3. (JPE, September 1996) The matrix

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$$

has eigenpairs

$$(\lambda, x) = \left(2, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right), \left(-1, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right), \left(3, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right),$$

Suppose the power method is applied with starting vector

$$z_0 = [1, 1, -1]^t/\sqrt{3}$$

(a) Determine whether or not the iteration will converge to an eigenpair of A, and if so, which one. Assume exact arithmetic.

(b) Repeat (a), except now use the inverse iteration with the same starting vector z_0 and the Rayleigh quotient of z_0 as approximation for the eigenvalue.

(c) Now answer both (a) and (b) again, except this time use standard fixed precision floating point arithmetic, i.e. computer arithmetic.