MA 660-1F, Dr Chernov

Assignment #2

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Due Mon, Jan 30

1. Let $A \in \mathbb{C}^{n\times n}$ satisfy $A^* = -A$. Show that the matrix $I - A$ is invertible. Then show that the matrix $(I - A)^{-1}(I + A)$ is unitary.

2. Let $A = (a_{ij})$ be a complex $n \times n$ matrix. Assume that $\langle Ax, x\rangle = 0$ for all $x \in \mathbb{C}^n$. Prove that
(a) $a_{ii} = 0$ for $1 \le i \le n$ by substituting $x = e_i$
(b) $a_{ij} = 0$ for $i \ne j$ by substituting $x = pe_i + qe_j$ then using (a) and putting $p, q = \pm 1, \pm \mathbf{i}$ (here $\mathbf{i} = \sqrt{-1}$) in various combinations.
Conclude that $A = 0$.

3. Find a real $n \times n$ matrix $A \ne 0$ such that $\langle Ax, x\rangle = 0$ for all $x \in \mathbb{R}^n$.

4. Find a real $n \times n$ matrix A such that $\langle Ax, x\rangle > 0$ for all $x \ne 0$, but A is not symmetric.