MA 660-2C, Dr Chernov

Warm-up homework assignment #3

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Due Tue, Sep 11

1. Problem 2.6 from Textbook.

2. The spectral radius of a real $n \times n$ matrix A is defined by

$$\rho(A) := \max\{|\lambda| : \ \lambda \text{ eigenvalue of } A\}$$

Show that
(a) $\rho(A) \le ||A||$ for every matrix norm $||\cdot||$ that is induced by a norm on $\mathbb{R}^n$
(b) if A is symmetric then $||A||_2 = \rho(A)$ (Hint: use the spectral theorem)
(c) $||A||_2^2 = \rho(A^t A)$

3. Let $A = (a_{ij})$ be a complex $n \times n$ matrix. Assume that $\langle Ax, x\rangle = 0$ for all $x \in \mathbb{C}^n$. Prove that
(a) $a_{ii} = 0$ for $1 \le i \le n$ by substituting $x = e_i$
(b) $a_{ij} = 0$ for $i \ne j$ by substituting $x = pe_i + qe_j$ then using (a) and putting $p, q = \pm 1, \pm i$ (here $i = \sqrt{-1}$) in various combinations
Conclude that $A = 0$.

4. Find a real $n \times n$ matrix $A \ne 0$ such that $\langle Ax, x\rangle = 0$ for all $x \in \mathbb{R}^n$.

5. Find a real $n \times n$ matrix A such that $\langle Ax, x\rangle > 0$ for all $x \ne 0$, but A is not symmetric. Hence, the symmetry requirement in Definition 12.9 cannot be dropped in the real case.