MA 660-3A, Dr Chernov — Assignment #2

. — Due Mon, Jan 28

1. Show that if a matrix A is both triangular and unitary, then it is diagonal.

2. Let $A \in \mathbb{C}^{n\times n}$ satisfy $A^* = -A$. Show that the matrix $I - A$ is invertible. Then show that the matrix $(I-A)^{-1}(I+A)$ is unitary.

3. Let $A = (a_{ij})$ be a complex $n \times n$ matrix. Assume that $\langle Ax, x\rangle = 0$ for all $x \in \mathbb{C}^n$. Prove that
(a) $a_{ii} = 0$ for $1 \le i \le n$ by substituting $x = e_i$
(b) $a_{ij} = 0$ for $i \neq j$ by substituting $x = pe_i + qe_j$ then using (a) and putting $p, q = \pm 1, \pm \mathbf{i}$ (here $\mathbf{i} = \sqrt{-1}$) in various combinations.
Conclude that $A = 0$.

4. Find a real $n \times n$ matrix $A \neq 0$ such that $\langle Ax, x\rangle = 0$ for all $x \in \mathbb{R}^n$.

5. Let $A \in \mathbb{C}^{m\times n}$. Show that

$$\|UA\|_F = \|AV\|_F = \|A\|_F$$

for any unitary matrices $U \in \mathbb{C}^{m\times m}$ and $V \in \mathbb{C}^{n\times n}$. Here $\|\cdot\|_F$ stands for the Frobenius norm.