1. Let $x \in \mathbb{C}^n$ and $y \in \mathbb{C}^m$. Consider the $m \times n$ matrix defined by $A = yx^*$. Show that rank $A = 1$. Show that $\|A\|_2 = \|x\|_2 \|y\|_2$. Show that $\|A\|_F = \|x\|_2 \|y\|_2$.

2. (JPE, September 1996) Compute the singular values of

$$A = \begin{pmatrix} 0 & -1.6 & 0.6 \\ 0 & 1.2 & 0.8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3. (JPE, May 2003) Determine the singular value decomposition for the matrix

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}$$

4. Let $Q \in \mathbb{C}^{n \times n}$ be unitary. Find all singular values of Q.

5. Show that if two matrices $A, B \in \mathbb{C}^{n \times n}$ are unitary equivalent, then they have the same singular values. Is the converse true?