MA 660-2C, Dr Chernov

Homework assignment #4

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Due Tue, Sep 18

1. (JPE, May 1994) Let $A \in \mathbb{R}^{n\times n}$ be given, symmetric and positive definite. Define $A_0 = A$, and consider the sequence of matrices defined by

$$A_k = G_k G_k^t \qquad \text{and} \qquad A_{k+1} = G_k^t G_k$$

where $A_k = G_k G_k^t$ is the Cholesky factorization for A_k. Prove that the A_k all have the same eigenvalues.

2. Let $A \in \mathbb{C}^{m\times n}$. Define $A^* := \bar{A}^t \in \mathbb{C}^{n\times m}$. Show that

$$\langle x, Ay\rangle = \langle A^* x, y\rangle$$

for all $x \in \mathbb{C}^m$ and $y \in \mathbb{C}^n$. (Note that A is a rectangular rather than a square matrix!)

3. For $A \in \mathbb{C}^{m\times n}$ define

$$||A||_2 := \sup_{||x||_2=1} ||Ax||_2$$

where $||x||_2$ is the 2-norm in $\mathbb{C}^n$ and $||Ax||_2$ is the 2-norm in $\mathbb{C}^m$. Prove that

$$||z||_2 = \sup_{||y||_2=1} |\langle y, z\rangle|$$

and

$$||A^* A||_2 = ||A||_2^2 = ||A^*||_2^2$$