1. Let $A \in \mathbb{C}^{n \times n}$ satisfy $A^* = -A$. Show that the matrix $I - A$ is invertible. Then show that the matrix $(I - A)^{-1}(I + A)$ is unitary.

2. (JPE, September 2002) Consider the matrix

$$A = \begin{pmatrix} -2 & 11 \\ -10 & 5 \end{pmatrix}$$

(a) Determine a real SVD of A.
(b) What are the 1-, 2-, ∞-, and Frobenius norm of A?
(c) Find A^{-1} not directly, but via the SVD.

3. Show that if two matrices $A, B \in \mathbb{C}^{n \times n}$ are unitary equivalent, then they have the same singular values. Is the converse true?

4. Find the numerical rank with tolerance 0.9 of the matrix

$$A = \begin{pmatrix} 3 & 2 \\ -4 & -5 \end{pmatrix}$$

5. (JPE, May 2003) Determine the singular value decomposition for the matrix

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}$$