MA 660-VT, Dr Chernov

Assignment #4

Due Wed, Feb 9

1. (JPE, September 1998). Show that diagonalizable complex matrices make a dense subset of $\mathbb{C}^{n\times n}$. That is, for any $A \in \mathbb{C}^{n\times n}$ and $\varepsilon > 0$ there is a diagonalizable $B \in \mathbb{C}^{n\times n}$ such that $\|A - B\|_2 < \varepsilon$. (Hint: use Schur decomposition theorem).

2. (JPE, September 2002) A matrix $A \in \mathbb{C}^{n\times n}$ is said to be *skew Hermitian* if $A^* = -A$.
(a) Prove that if A is skew Hermitian and B is unitary equivalent to A, then B is also skew Hermitian.
(b) What special form does the Schur decomposition take for a skew Hermitian matrix A?
(c) Prove that the eigenvalues of a skew Hermitian matrix are purely imaginary, i.e. they satisfy $\bar{\lambda} = -\lambda$.

3. Prove that for any $A \in \mathbb{C}^{n\times n}$

$$\lim_{n\to\infty} \|A^n\| = 0 \quad \Longleftrightarrow \quad \rho(A) < 1$$

where $\|\cdot\|$ is any matrix norm induced by a vector norm. Hint: use Jordan decomposition.