MA 660-1D, Dr Chernov — Homework assignment #5
. — Due Mon, Feb 16

1. Prove properties 4, 7, 9, 10 of condition numbers listed in Proposition 15.6.

2. (JPE, September 1992) Let A and $A+E$ be nonsingular, and

$$Ax = b, \qquad (A+E)x_c = b$$

where b is non-zero. Prove directly that

$$\frac{\|x - x_c\|_\infty}{\|x_c\|_\infty} \le \kappa_\infty(A) \frac{\|E\|_\infty}{\|A\|_\infty}$$

Next, let

$$A = \begin{pmatrix} 4.1 & 2.8 \\ 9.7 & 6.6 \end{pmatrix}, \quad E = \begin{pmatrix} 0.9 & 0.2 \\ 0.3 & 0.4 \end{pmatrix}, \quad x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Use the estimate obtained above to find a lower bound for $\kappa_\infty(A)$. Check this estimate by computing $\kappa_\infty(A)$ directly.

3. (JPE, May 1997). Compute the condition numbers κ_1, κ_2 and κ_∞ for the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 1.01 & 2 \end{pmatrix}$$

4. (JPE, May 1992). Compute the condition number κ_∞ for the matrix

$$A_n = \begin{pmatrix} 1 & 2 \\ 2 & 4 + n^{-2} \end{pmatrix}$$

Now, suppose that the systems $A_n x = b$, $n = 1, 2, \ldots$, are being solved for some $b \in \mathbb{R}^2$ on a computer employing binary floating point arithmetic with a 23 digit mantissa, and using chopped arithmetic. For which values of n can the computed solution be trusted? (Hint: first write down the unit roundoff (machine precision) $\mathbf{u}$).