1. (JPE, September 2002) Consider the matrix

$$A = \begin{pmatrix} -2 & 11 \\ -10 & 5 \end{pmatrix}$$

(a) Determine a real SVD of $A$.
(b) What are the 1-, 2-, $\infty$-, and Frobenius norm of $A$?
(c) Find $A^{-1}$ not directly, but via the SVD.

2. Show that if two matrices $A, B \in \mathbb{C}^{n \times n}$ are unitary equivalent, then they have the same singular values. Is the converse true?

3. Prove that full rank matrices make an open subset of $\mathbb{R}^{m \times n}$.

4. Find the numerical rank with tolerance 0.9 of the matrix

$$A = \begin{pmatrix} 3 & 2 \\ -4 & -5 \end{pmatrix}$$

5. Let $Q \in \mathbb{C}^{n \times n}$ be unitary. Find all singular values of $Q$.

Bonus. (JPE, May 2003) Determine the singular value decomposition for the matrix

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}$$