MA 660-VT, Dr Chernov | Assignment #5
. | Due Wed, Feb 16

1. (combined from JPE, October 1990 and May 1997) Let $A \in \mathbb{C}^{n\times n}$ be a normal matrix.
(a) Prove that $A - \lambda I$ is normal for any $\lambda \in \mathbb{C}$.
(b) Prove that $\|Ax\|_2 = \|A^*x\|_2$ for all x.
(c) Prove that (λ, x) is an eigenpair of A if and only if $(\bar{\lambda}, x)$ is an eigenpair of A^*. (Hence, A and A^* have the same eigenvectors.)

2. (JPE, May 1996). Let T be a linear operator on a finite dimensional complex inner product space V, and let T^* be the adjoint of T. Prove that $T = T^*$ if and only if $T^*T = T^2$. (Note: this is a difficult problem! Hint: reduce the problem to complex matrices and use the Schur decomposition.)

3. Find a nonzero matrix $A \in \mathbb{R}^{2\times 2}$ that admits at least two LU decomposition, i.e. $A = L_1U_1 = L_2U_2$, where L_1 and L_2 are two *distinct* unit lower triangular matrices and U_1 and U_2 are two *distinct* upper triangular matrices.

4. Show that the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ admits no LU decomposition, even if we only require that L be lower triangular (not necessarily unit lower triangular).