MA 660-2C, Dr Chernov
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Homework assignment #6
Due Tue, Oct 2

1. Show that if $S \in \mathbb{R}^{n\times n}$ is a singular matrix, then $||I - S|| \geq 1$. Let $A \in \mathbb{R}^{n\times n}$ be a nonsingular matrix and $B \in \mathbb{R}^{n\times n}$ a singular matrix. Prove that

$$\frac{1}{\kappa(A)} \leq \frac{||A - B||}{||A||}$$

Here $||\cdot||$ is a matrix norm induced by some norm in $\mathbb{R}^n$, and $\kappa(A)$ is the condition number of A with respect to the norm $||\cdot||$.

2. Prove properties 1 to 10 of condition numbers listed in Proposition 15.6.

3. (JPE, September 1992) Let A and $A + E$ be nonsingular, and

$$Ax = b, \qquad (A + E)x_c = b$$

where b is non-zero. Prove directly that

$$\frac{||x - x_c||_\infty}{||x_c||_\infty} \leq \kappa_\infty(A) \frac{||E||_\infty}{||A||_\infty}$$

Next, let

$$A = \begin{pmatrix} 4.1 & 2.8 \\ 9.7 & 6.6 \end{pmatrix}, \qquad E = \begin{pmatrix} 0.9 & 0.2 \\ 0.3 & 0.4 \end{pmatrix}, \qquad x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Use the estimate obtained above to find a lower bound for $\kappa_\infty(A)$. Check this estimate by computing $\kappa_\infty(A)$ directly.