MA 660-1D, Dr Chernov | Homework assignment #6
. | Due Mon, Feb 23

1. Let $x, y \in \mathbb{R}^n$ be such that $x \neq y$ and $||x||_2 = ||y||_2 \neq 0$. Show that there is a unique reflector matrix P such that $Px = y$. Is this true for vectors $x, y \in \mathbb{C}^n$?

2. (JPE, September 1997) Let

$$A = \begin{pmatrix} 3 & 3 \\ 0 & 4 \\ 4 & -1 \end{pmatrix}, \quad \text{and} \quad b = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

Use the Gram-Schmidt process to find an orthonormal basis for the column space of A. Factor A into a product QR where $Q \in \mathbb{R}^{3\times 2}$ has an orthonormal set of column vectors and $R \in \mathbb{R}^{2\times 2}$ is upper triangular. Solve the least squares problem $Ax = b$. Compute the norm of the residual vector, $\|r\|$.

3. (JPE, May 1998). Given the data (0,1), (3,4) and (6,5), use a QR factorization technique to find the best least squares fit by a linear function. Also, solve the problem via the system of normal equations.

4. (JPE, September 1997) Let $A \in \mathbb{C}^{n\times n}$ be nonsingular. Let $A = Q_1 R_1$ be a QR decomposition of A and for $k \geq 1$ define inductively $AQ_k = Q_{k+1} R_{k+1}$ a QR decomposition of AQ_k. Prove that there exists an upper triangular matrix U_k such that $Q_k = A^k U_k$ and a lower triangular matrix L_k such that $Q_k = (A^*)^{-k} L_k$.

[Bonus] Problem 7.5 (b) from the textbook.