MA 660-3A, Dr Chernov

Assignment #6

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Due Mon, Feb 22

1. (JPE, May 1994) Let $A \in \mathbb{R}^{n\times n}$ be given, symmetric and positive definite. Define $A_0 = A$, and consider the sequence of matrices defined by

$$A_k = G_k G_k^T \qquad \text{and} \qquad A_{k+1} = G_k^T G_k$$

where $A_k = G_k G_k^T$ is the Cholesky factorization for A_k. Prove that the A_k all have the same eigenvalues.

2. Find a nonzero matrix $A \in \mathbb{R}^{2\times 2}$ that admits at least two LU decomposition, i.e. $A = L_1 U_1 = L_2 U_2$, where L_1 and L_2 are two *distinct* unit lower triangular matrices and U_1 and U_2 are two *distinct* upper triangular matrices.

3. Show that the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ admits no LU decomposition, even if we only require that L be lower triangular (not necessarily unit lower triangular).