MA 660-2C, Dr Chernov — Homework assignment #7

. — Due Tue, Oct 9

1. (JPE, May 1997). Compute the condition numbers κ_1, κ_2 and κ_∞ for the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 1.01 & 2 \end{pmatrix}$$

2. (JPE, May 1992). Compute the condition number κ_∞ for the matrix

$$A_n = \begin{pmatrix} 1 & 2 \\ 2 & 4 + n^{-2} \end{pmatrix}$$

Now, suppose that the systems $A_n x = b$, $n = 1, 2, \ldots$, are being solved for some $b \in \mathbb{R}^2$ on a computer employing binary floating point arithmetic with a 23 digit mantissa, and using chopped arithmetic. For which values of n can the computed solution be trusted? (Hint: first write down the unit roundoff (machine precision) $\mathbf{u}$).

3. Let $x, y \in \mathbb{R}^n$ be such that $x \neq y$ and $||x||_2 = ||y||_2 \neq 0$. Show that there is a unique reflector matrix P such that $Px = y$. Is this true for vectors $x, y \in \mathbb{C}^n$?