MA 660-1D, Dr Chernov

Homework assignment #7

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Due Mon, Mar 8

1. (JPE, September 2002) A matrix $A \in \mathbb{C}^{n\times n}$ is said to be *skew Hermitian* if $A^* = -A$.
(a) Prove that if A is skew Hermitian and B is unitary equivalent to A, then B is also skew Hermitian.
(b) What special form does the Schur decomposition take for a skew Hermitian matrix A?
(c) Prove that the eigenvalues of a skew Hermitian matrix are purely imaginary, i.e. they satisfy $\bar{\lambda} = -\lambda$.

2. (combined from JPE, October 1990 and May 1997) Let $A \in \mathbb{C}^{n\times n}$ be a normal matrix.
(a) Prove that $A - \lambda I$ is normal for any $\lambda \in \mathbb{C}$.
(b) Prove that $\|Ax\| = \|A^*x\|$ for all x.
(c) Prove that (λ, x) is an eigenpair of A if and only if $(\bar{\lambda}, x)$ is an eigenpair of A^*. (Hence, A and A^* have the same eigenvectors.)

3. (JPE, September 2002) Consider the matrix

$$A = \begin{pmatrix} -2 & 11 \\ -10 & 5 \end{pmatrix}$$

(a) Determine a real SVD of A.
(b) What are the 1-, 2-, ∞-, and Frobenius norm of A?
(c) Find A^{-1} not directly, but via the SVD.

4. (JPE, May 2003) Determine the singular value decomposition for the matrix

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}$$

[Bonus] (JPE, May 1996). Let T be a linear operator on a finite dimensional complex inner product space V, and let T^* be the adjoint of T. Prove that $T = T^*$ if and only if $T^*T = T^2$. (Hint: reduce the problem to complex matrices and use the Schur decomposition.)