MA 660-3A, Dr Chernov | Assignment #7
. | Due Mon, Feb 25

1. (JPE, May 1994) Let $A \in \mathbb{R}^{n\times n}$ be given, symmetric and positive definite. Define $A_0 = A$, and consider the sequence of matrices defined by

$$A_k = G_k G_k^T \qquad \text{and} \qquad A_{k+1} = G_k^T G_k$$

where $A_k = G_k G_k^T$ is the Cholesky factorization for A_k. Prove that the A_k all have the same eigenvalues.

2. Find a nonzero matrix $A \in \mathbb{R}^{2\times 2}$ that admits at least two LU decomposition, i.e. $A = L_1U_1 = L_2U_2$, where L_1 and L_2 are two *distinct* unit lower triangular matrices and U_1 and U_2 are two *distinct* upper triangular matrices.

4. (JPE, September 2002) Consider three vectors

$$v_1 = \begin{pmatrix} 1 \\ \epsilon \\ 0 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 0 \\ \epsilon \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \epsilon \end{pmatrix}.$$

where $\epsilon \ll 1$.

(a) Use the classical Gram-Schmidt method to compute 3 **orthonormal** vectors q_1, q_2, q_3, making the approximation that $1 + \epsilon^2 \approx 1$ (that is, replace any term containing ϵ^2 or smaller with zero, **but** retain terms containing ϵ). Are q_i $(i = 1, 2, 3)$ pairwise orthogonal? If not, why not?

(b) Repeat (a) using the modified Gram-Schmidt orthogonalization process. Are the q_i $(i = 1, 2, 3)$ pairwise orthogonal? If not, why not?

5. (Bonus problem) Show that the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ admits no LU decomposition, even if we only require that L be lower triangular (not necessarily unit lower triangular).