MA 660-3A, Dr Chernov

Assignment #7
Due Mon, Mar 1

1. (JPE, September 2002) Consider three vectors

$$v_1 = \begin{pmatrix} 1 \\ \epsilon \\ 0 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 0 \\ \epsilon \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \epsilon \end{pmatrix}.$$

where $\epsilon \ll 1$.

(a) Use the classical Gram-Schmidt method to compute 3 **orthonormal** vectors q_1, q_2, q_3, making the approximation that $1 + \epsilon^2 \approx 1$ (that is, replace any term containing ϵ^2 or smaller with zero, **but** retain terms containing ϵ). Are q_i $(i = 1, 2, 3)$ pairwise orthogonal? If not, why not?

(b) Repeat (a) using the modified Gram-Schmidt orthogonalization process. Are the q_i $(i = 1, 2, 3)$ pairwise orthogonal? If not, why not?

2. (JPE, September 1997) Let

$$A = \begin{bmatrix} 3 & 3 \\ 0 & 4 \\ 4 & -1 \end{bmatrix}, \quad \text{and} \quad b = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

Use the Gram-Schmidt process to find an orthonormal basis for the column space of A. Factor A into a product QR where $Q \in \mathbb{R}^{3\times 2}$ has an orthonormal set of column vectors and $R \in \mathbb{R}^{2\times 2}$ is upper triangular. Solve the least squares problem $Ax = b$. Compute the norm of the residual vector, $\|r\|$.

3. (JPE, May 1998). Given the data (0,1), (3,4) and (6,5), use a QR factorization technique to find the best least squares fit by a linear function. Also, solve the problem via the system of normal equations.