MA 660-VT, Dr Chernov — Assignment #7

. Due Wed, Mar 2

1. (JPE, September 1997) Let

$$A = \begin{bmatrix} 3 & 3 \\ 0 & 4 \\ 4 & -1 \end{bmatrix}, \quad \text{and} \quad b = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

Use the Gram-Schmidt process to find an orthonormal basis for the column space of A. Factor A into a product QR where $Q \in \mathbb{R}^{3\times 2}$ has an orthonormal set of column vectors and $R \in \mathbb{R}^{2\times 2}$ is upper triangular. Solve the least squares problem $Ax = b$. Compute the norm of the residual vector, $\|r\|$.

2. (JPE, May 1998). Given the data (0,1), (3,4) and (6,5), use a QR factorization technique to find the best least squares fit by a linear function. Also, solve the problem via the system of normal equations.

3. (JPE, September 2009). Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -2 \\ 1 & 3 \\ 1 & 0 \end{bmatrix}$$

(a) Find a reduced QR factorization of A by the Gram-Schmidt process.

(b) Use the QR factorization from (a) to find the least squares fit by a linear function for $(1, -2)$, $(-2, 0)$, $(3, 2)$ and $(0, 3)$.

4. (JPE, May 2008). Consider the linear least squares problem $\min_x \|Ax - b\|_2^2$ with

$$A = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix}$$

(a) Solve the above least squares problem using normal equations.

(b) Compute the reduced singular value decomposition of A, then use it to solve the above least squares problem.