MA 660-1F, Dr Chernov — Assignment #7

. — Due Mon, Mar 5

1. (JPE, September 1993). Solve the system

$$\begin{pmatrix} 0.001 & 1.00 \\ 1.00 & 2.00 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1.00 \\ 3.00 \end{pmatrix}$$

using the LU decomposition with and without partial pivoting and chopped arithmetic with base $\beta = 10$ and $t = 3$ (i.e., work with a three digit mantissa). Obtain computed solutions (x_c, y_c) in both cases. Find the exact solution, compare, make comments.

2. (JPE, May 2003). Consider the system

$$\begin{pmatrix} \varepsilon & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Assume that $|\varepsilon| \ll 1$. Solve the system by using the LU decomposition with and without partial pivoting and adopting the following rounding off models (at all stages of the computation!):

$$a + b\varepsilon = a \qquad \text{(for } a \neq 0\text{)},$$
$$a + b/\varepsilon = b/\varepsilon \qquad \text{(for } b \neq 0\text{)}.$$

Find the exact solution, compare, make comments.

3. (JPE, September 1997). Show that, given a matrix $A \in \mathbb{R}^{n\times n}$, one can choose vectors b and Δb so that if

$$Ax = b$$
$$A(x + \Delta x) = b + \Delta b$$

then

$$\frac{||\Delta x||_2}{||x||_2} = \kappa_2(A)\frac{||\Delta b||_2}{||b||_2}$$

Explain the significance of this result for the 'optimal' role of condition numbers in the sensitivity analysis of linear systems.
(Hint: use the SVD theorem to show that it is enough to consider the case where A is a diagonal matrix.)

4. (JPE, combined May 1997 and May 2008)
(a) Compute the condition numbers κ_1, κ_2 and κ_∞ for the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 1.01 & 2 \end{pmatrix}$$

(b) Show that for every non-singular 2×2 matrix A we have $\kappa_1(A) = \kappa_\infty(A)$.