

1. Prove that full rank matrices make an open subset of $\mathbb{R}^{m \times n}$.
2. Find the numerical rank with tolerance 0.9 of the matrix

$$A = \begin{pmatrix} 3 & 2 \\ -4 & -5 \end{pmatrix}$$

3. (JPE, September 1998). Show that diagonalizable (complex) matrices make a dense subset of $\mathbb{C}^{n \times n}$. That is, for any $A \in \mathbb{C}^{n \times n}$ and $\varepsilon > 0$ there is a diagonalizable $B \in \mathbb{C}^{n \times n}$ such that $\|A - B\|_2 < \varepsilon$. (Hint: use Schur decomposition theorem).