MA 660-3A, Dr Chernov — Assignment #8

. — Due Tue, Mar 4

1. (JPE, September 1997) Let

$$A = \begin{bmatrix} 3 & 3 \\ 0 & 4 \\ 4 & -1 \end{bmatrix}, \quad \text{and} \quad b = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

Use the Gram-Schmidt process to find an orthonormal basis for the column space of A. Factor A into a product QR where $Q \in \mathbb{R}^{3\times 2}$ has an orthonormal set of column vectors and $R \in \mathbb{R}^{2\times 2}$ is upper triangular. Solve the least squares problem $Ax = b$. Compute the norm of the residual vector, $\|r\|$.

2. (JPE, May 1998). Given the data (0,1), (3,4) and (6,5), use a QR factorization technique to find the best least squares fit by a linear function. Also, solve the problem via the system of normal equations.

3. (JPE, May 2000). Use a QR decomposition, with exact arithmetic, to solve the least squares problem for the overdetermined system

$$A = \begin{bmatrix} 1 & -3 \\ 2 & 4 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ -5 \end{bmatrix}$$

State the magnitude of the minimum residual.