

1. (JPE, September 1993). Solve the system

$$\begin{pmatrix} 0.001 & 1.00 \\ 1.00 & 2.00 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1.00 \\ 3.00 \end{pmatrix}$$

using the LU decomposition with and without partial pivoting and chopped arithmetic with base $\beta = 10$ and $t = 3$ (i.e., work with a three digit mantissa). Obtain computed solutions (x_c, y_c) in both cases. Find the exact solution, compare, make comments.

2. (JPE, May 2003). Consider the system

$$\begin{pmatrix} \varepsilon & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Assume that $|\varepsilon| \ll 1$. Solve the system by using the LU decomposition with and without partial pivoting and adopting the following rounding off models (at all stages of the computation!):

$$\begin{aligned} a + b\varepsilon &= a && (\text{for } a \neq 0), \\ a + b/\varepsilon &= b/\varepsilon && (\text{for } b \neq 0). \end{aligned}$$

Find the exact solution, compare, make comments.