MA 660-1F, Dr Chernov

Assignment #8

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Due Mon, Mar 12

1. (JPE, September 2002). Consider a linear system $Ax = b$. Let x^* be the exact solution, and let x_c be some computed approximate solution. Let $e = x^* - x_c$ be the error and $r = b - Ax_c$ the residual for x_c. Show that

$$\frac{1}{\kappa(A)} \frac{\|r\|}{\|b\|} \le \frac{\|e\|}{\|x^*\|} \le \kappa(A) \frac{\|r\|}{\|b\|}$$

Interpret the above inequality for $\kappa(A)$ close to 1 and for $\kappa(A)$ large.

2. Prove properties 7 and 8 of condition numbers listed in Proposition 12.8.

3. Suppose the condition number of a rectangular matrix $A \in \mathbb{C}^{m\times n}$ with $m > n$ is defined by

$$\kappa(A) \colon = \frac{\sup_{\|x\|=1} \|Ax\|}{\inf_{\|x\|=1} \|Ax\|}.$$

Prove that

$$[\kappa_2(A)]^2 = \kappa_2(A^*A) = \frac{\lambda_{\max}(A^*A)}{\lambda_{\min}(A^*A)}$$

4. Let $x, y \in \mathbb{C}^n$ be such that $x \neq y$ and $\|x\|_2 = \|y\|_2 \neq 0$. Show that there is a reflector matrix P such that $Px = y$ if and only if $\langle x, y\rangle \in \mathbb{R}$. For an extra credit: show that if the above reflector exists, then it is unique.