MA 660-2C, Dr Chernov | Homework assignment #9
. | Due Tue, Oct 23

1. (JPE, September 1997) Let $A \in \mathbb{C}^{n\times n}$ be nonsingular. Let $A = Q_1 R_1$ be a QR decomposition of $A$ and for $k \geq 1$ define inductively $AQ_k = Q_{k+1}R_{k+1}$ a QR decomposition of $AQ_k$. Prove that there exists an upper triangular matrix $U_k$ such that $Q_k = A^k U_k$ and a lower triangular matrix $L_k$ such that $Q_k = (A^*)^{-k} L_k$.

2. (combined from JPE, October 1990 and May 1997) Let $A \in \mathbb{C}^{n\times n}$ be a normal matrix.
(a) Prove that $A - \lambda I$ is normal for any $\lambda \in \mathbb{C}$. Prove that $||Ax|| = ||A^*x||$ for all $x$.
(b) Prove that $(\lambda, x)$ is an eigenpair of $A$ if and only if $(\bar{\lambda}, x)$ is an eigenpair of $A^*$. (Hence, $A$ and $A^*$ have the same eigenvectors.)

3. (JPE, May 1996). Let $T$ be a linear operator on a finite dimensional complex inner product space $V$, and let $T^*$ be the adjoint of $T$. Prove that $T = T^*$ if and only if $T^*T = T^2$. (Hint: reduce the problem to complex matrices and use the Schur decomposition.)