Assignment #9 Due Tue, Mar 18

1. (JPE, September 1993). Solve the system

$$\left(\begin{array}{cc} 0.001 & 1.00\\ 1.00 & 2.00 \end{array}\right) \left(\begin{array}{c} x\\ y \end{array}\right) = \left(\begin{array}{c} 1.00\\ 3.00 \end{array}\right)$$

using the LU decomposition with and without partial pivoting and chopped arithmetic with base  $\beta = 10$  and t = 3 (i.e., work with a three digit mantissa). Obtain computed solutions  $(x_c, y_c)$  in both cases. Find the exact solution, compare, make comments.

2. (JPE, May 2003). Consider the system

$$\left(\begin{array}{cc}\varepsilon & 1\\ 2 & 1\end{array}\right)\left(\begin{array}{c}x\\y\end{array}\right) = \left(\begin{array}{c}1\\0\end{array}\right)$$

Assume that  $|\varepsilon| \ll 1$ . Solve the system by using the LU decomposition with and without partial pivoting and adopting the following rounding off models (at all stages of the computation!):

$$\begin{aligned} a + b\varepsilon &= a \quad (\text{for } a \neq 0), \\ a + b/\varepsilon &= b/\varepsilon \quad (\text{for } b \neq 0). \end{aligned}$$

Find the exact solution, compare, make comments.

3. (JPE, September 1997). Show that, given a matrix  $A \in \mathbb{R}^{n \times n}$ , one can choose vectors b and  $\Delta b$  so that if Ax = b

$$A(x + \Delta x) = b + \Delta b$$

then

$$\frac{||\Delta x||_2}{||x||_2} = \kappa_2(A) \frac{||\Delta b||_2}{||b||_2}$$

Explain the significance of this result for the 'optimal' role of condition numbers in the sensitivity analysis of linear systems.

(Hint: use the SVD theorem to show that it is enough to consider the case where A is a diagonal matrix.)