MA 660-1F, Dr Chernov | Assignment #9
. | Due Mon, Apr 2

1. (JPE May, 1994). Let $X^{-1}AX = D$, where D is a diagonal matrix.
(i) Show that the columns of X are right eigenvectors and the conjugate rows of X^{-1} are left eigenvectors of A.
(ii) Let $\lambda_1 \dots, \lambda_n$ be the eigenvalues of A. Show that there are right eigenvectors $x_1, \dots, x_n$ and left eigenvectors $y_1, \dots, y_n$ such that

$$A = \sum_{i=1}^{n} \lambda_i x_i y_i^*$$

2. Let $A \in \mathbb{C}^{n\times n}$ be Hermitean with eigenvalues $\lambda_1 \le \cdots \le \lambda_n$. Let $\mu_1 \le \cdots \le \mu_{n-1}$ be all the eigenvalues of the $(n-1)$-st principal minor A_{n-1} of A. Use the Minimax theorem to prove the *interlacing property*

$$\lambda_1 \le \mu_1 \le \lambda_2 \le \cdots \le \lambda_{n-1} \le \mu_{n-1} \le \lambda_n$$

3. Let $A \in \mathbb{C}^{n\times n}$. Show that
(i) λ is an eigenvalue of A iff $\bar{\lambda}$ is an eigenvalue of A^*.
(ii) if A is normal, then for each eigenvalue the left and right eigenspaces coincide;
(iii) if A is normal, then for any simple eigenvalue λ of A we have $K(\lambda) = 1$.

4. Let $A \in \mathbb{C}^{n\times n}$ and $B = Q^*AQ$, where Q is a unitary matrix. Show that if the left and right eigenspaces of A are equal, then B enjoys the same property. After that show that A is normal. Conclude that if A has all simple eigenvalues with $K(\lambda) = 1$, then A is normal.

5. If λ is an eigenvalue of geometric multiplicity ≥ 2 for a matrix A, show that for each right eigenvector x there is a left eigenvector y such that $y^*x = 0$.