Abstracts for Talks

(The abstract for the distinguished lectures may be included in a separate document.)

Sven Bachmann, LMU München.

Classifying quantum phases.

A first step in the understanding of ground state phases of quantum spin systems and the transitions between them is to develop a physically meaningful and mathematically precise classification. Using the notion of local unitary equivalence and its generalization to automorphic equivalence, I will present a classification in one dimension that emphasizes the role of edge states. I will also describe how local symmetries enter the picture, and how associated invariants lead to a finer classification. This is joint work with B. Nachtergaele and Y. Ogata

Spyridon Michalakis, Institute for Quantum Information at Caltech.

Quasi-adiabatic evolution: A powerful new tool in mathematical physics.

The introduction by Hastings, ten years ago, of a novel way to transport states under a family of gapped Hamiltonians, has had a tremendous impact on mathematical physics since then. The new evolution, called quasi-adiabatic continuation, enjoys the powerful properties of the well-known adiabatic evolution of Kato, but has a crucial advantage: When the energy gap between the subspace of states being evolved and the rest of the spectrum becomes vanishingly small, the generators of the evolution remain quasi-local due to Lieb-Robinson bounds that are oblivious to energy gaps. In contrast, adiabatic evolution generates non-local changes near these critical points, forced to avoid energy crossings at all costs. I will introduce the latest version of quasi-adiabatic evolution, prove that it simulates exactly adiabatic evolution when a spectral gap exists, and show that its generators are always quasi-local. If time permits, I will discuss some of the applications of this new tool in resolving long-standing questions in mathematical physics.

Pieter Naaijkens, Leibniz Universität Hannover.

Superselection sectors in quantum spin systems.

In certain quantum mechanical systems one can build superpositions of states whose relative phase is not observable. This is related to superselection sectors: the algebra of observables in such a situation acts as a direct sum of irreducible representations on a Hilbert space. Physically, this implies that there are certain global quantities that one cannot change with local operations, for example the total charge of the system.

Here I will discuss how superselection sectors arise in quantum spin systems, and how one can deal with them mathematically. As an example we apply some of these ideas to Kitaev's toric code model, to show how the analysis of the superselection sectors can be used to get a complete understanding of the "excitations" or "charges" in this model. In particular one can show that these excitations are so-called anyons.

Alireza Rafiyi, Memorial University of Newfoundland.

Mean field limit in open quantum systems.

I will shortly speak about mean-field limit in quantum systems by reviewing the work of Herbert Spohn for lattice systems. Then, I give a sketch of my Ph.D. research project on the mean field limit in open quantum systems.

Robert Sims, University of Arizona.

Lieb-Robinson Bounds: A Tutorial

An important tool in the analysis of quantum spin systems is the so-called Lieb-Robinson bound. These estimates demonstrate a quasi-locality for the dynamics associated to a general class of models, including many that commonly appear in the physics literature. In this tutorial, I will review a statement and proof of a Lieb-Robinson bound. As a simple application, I will prove the existence of a dynamics in the thermodynamic limit for models satisfying this Lieb-Robinson bound.

On the Random XY Spin Chain.

Building on the talk of Gunter Stolz, I will discuss dynamical localization, expressed in terms of a zero-velocity Lieb-Robinson bound, for a random XY spin chain. In the specific chain under consideration, the effective one-particle Hamiltonian is easily seen to be the well-studied Anderson model. As a consequence, a number of results from the single-particle theory of localization readily apply. A main goal of this talk is to illustrate how the many-body dynamics can be estimated in terms of the single-particle dynamics by 'unraveling' the Jordan-Wigner transformation.

Shannon Starr, UAB.

Tutorial on SU(2) *and spin waves.*

This is a tutorial to cover some basics about the representations of SU(2) and spin waves. The most important material, which will come up in other lectures is about SU(2). I will basically cover material about the representations of SU(2) which are often covered in an upper-division physics QM class when talking about the symmetries of the spin. So if you already know this, you may feel free to skip the tutorial, without fear of missing much.

I will describe the generators of SU(2) and the finite dimensional irreducible representations. I will describe the formulas for the tensor product of two irreducible representations. I will describe the Casimir operator, which is defined to be the generator of the center of the representation. The Casimir operator is just the total spin.

After this is done, I will do some easy calculations for the Heisenberg ferromagnet to introduce spin waves. I will not do the hard calculations. I will do easy calculations, such as lead to variational upper bounds on the free energy. Then I will state a bound of Correggi, Giuliani and Seiringer, which for low temperatures verifies the spin wave heuristic. They have shown lower bounds which match the upper bounds asymptotically as T goes to 0.

Gunter Stolz, UAB.

Tutorial on self-adjoint operators and solving the Schrödinger equation.

In this tutorial-style lecture we collect basic facts from the theory of self-adjoint operators, mostly with a view of what is relevant for applications in mathematical quantum mechanics, in particular for solving the Schrödinger equation. Specific topics include the spectral theorem and functional calculus for self-adjoint operators, Stone's Theorem, Laplacians and Fourier transform, Duhamel's formula and Dyson series, time-evolution of non-interacting quantum systems, as well as the Baker-Campbell-Hausdorff fomula and Trotter product formula. For proofs and further background on functional analysis and linear operator theory we refer to standard texts.

XY spin chain and Jordan-Wigner transform

We present an introduction to the mathematical theory of the XY spin chain. The importance of this model lies in the fact, first understood by Lieb, Schultz and Mattis in 1961, that the XY spin chain is one of very few exactly solvable models in the theory of quantum many-body systems. Lieb, Schultz and Mattis considered the constant coefficient case. In the variable coefficient case discussed here, "exactly solvable" should be understood as "reducible to an effective one-particle Hamiltonian". The key method behind this is the Jordan-Wigner transform, which allows to map the XY chain to a free Fermion system.

Daniel Ueltschi, University of Warwick.

Graphical representations for quantum spin systems.

There exists a class of quantum spin systems where quantum correlations can be expressed using random loop correlations. Included are the spin 1/2 Heisenberg ferro- and antiferromagnets, the XY model, and spin 1 models with SU(2) invariant interactions. In these lectures, I will introduce the probability models and prove their equivalence with quantum spins systems. I will discuss some properties and consequences of the loop representations.

Anna Vershynina, Institute for Quantum Information at the RWTH, Aachen.

A complete criterion for convex-Gaussian states detection.

It has been shown that the quantum computation through braiding of Majorana fermions supplemented by noisy ancillae can be efficiently simulated classically when the ancilla is a convex mixture of Gaussian fermionic states. I will present a complete criterion to determine whether a state is a convex mixture of Gaussian states. The criterion is based on the existence of a Gaussian-symmetric extension.

Amanda Young, U. C. Davis.

Spectral Gap of d-dimensional PVBS Models.

We introduce the class of the single species PVBS models defined on a d-dimensional hypercubic lattice. Using the martingale method, we determine the range of parameters for which spectral gap does not vanish in the thermodynamic limit. In two dimensions we proved an explicit lower bound for the gap. Furthermore, the model parameters for which the gap estimate vanishes are exactly those where we can show that the gap in fact vanishes in the thermodynamic limit.