MA 435/535

Name (Print last name first): . . . .

Final

Let H be a subgroup of G (with operation $*$) and let S be the set of all right cosets of H (the elements of S have the form $H * g$ for some $g \in G$). Define an operation $\circ$ on S by:

$$H * a \;\circ\; H * b = H * (a * b).$$

Question 1

Show with an example that $\circ$ is not necessarily a binary operation on S.

Question 2

Show that if H has the property

$$\text{for every } h \in H \text{ and } g \in G, \quad g * h * g^{-1} \in H,$$

then $g * H = H * g$ and consequently $\circ$ is a binary operation (you may assume all our results about right cosets hold analogously true for left cosets). By the way, if this property holds true for H, we call H a *normal* subgroup of G.

Question 3

Show that when H is a normal subgroup of G,

- The binary operation $\circ$ on S is associative.
- $H * e \;\circ\; H * g = H * g$ (existence of an identity).
- $H * g \;\circ\; H * g^{-1} = H * e$ (existence of inverses).

Thus S is a group under $\circ$. [S is called the *quotient group* G/H].

Question 4

In Problem 67 we saw when the elements a and c are as shown on page 23 of the text, then

$$S_3 = sg\{a, c | a^3 = e = c^2, ca = a^2c\}.$$

Let S_4 be the set of permutations of $\{1, 2, 3, 4\}$. Let a be the permutation that switches 1 and 2 and leaves 3 and 4 as is [we write $a = (12)$] and let b be the permutation that takes 1 to 2, 2 to 3, 3 to 4, and 4 back to 1 [we write $b = (1234)$]. Show that

$$S_4 = \{a, b | ???\}.$$

In other words, a and b are generators of S_4, but what are the defining relations?